Eur. Phys. J. B **20**, 543–546 (2001)

THE EUROPEAN
PHYSICAL JOURNAL B
EDP Sciences

Wavelet-based detection of coherent structures and self-affinity in financial data

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Received 5 September 2000

Abstract. As a linear superposition of translated and dilated versions of a chosen analyzing wavelet function, the wavelet transform lends itself to the analysis of underlying multi-scale structure in nonstationary time series. In this work, we use the discrete wavelet transform (DWT) to investigate scaling and search for the presence of coherent structures in financial data. Quantitative measurements are given by the DWT of the original time series and wavelet coefficient variance. We find that variations and correlations in the transform coefficients are able to indicate the presence of structure and that measurements based on the DWT allow us to observe scaling directly in the nonstationary time series.

PACS. 05.45.Tp Time series analysis – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion. – 02.90.+p Other topics in mathematical methods in physics.

1 Introduction

Recent work has shown that wavelet analysis [1] provides a unifying framework for the description of many time series phenomena in the examination of noise processes, chaotic signals and coherent structures [2]. Techniques derived from the Fourier decomposition of a given time series into a linear superposition of cosine functions of infinite support[1] are tools for global analysis and are subject to stationarity requirements. In comparison, due to the finite support of the analysis functions, the wavelet transform allows exploration of local data features directly in nonstationary data. The discrete wavelet transform (DWT) utilizes dilated and translated versions of a pre-specified *wavelet function* to probe structure on different time-scales. The self-affine nature of this construction has already been demonstrated to be a useful tool in studying fractal signals [3]. In this work, we use the discrete wavelet transform to obtain a set of *wavelet coefficients* that represent time series fluctuations on different time-scales. The log-variance plot of the coefficients as a function of some time-scale index delineates data self-affinity. Variations in this plot and correlations between wavelet coefficients are used to look for coherent structures. These structures are located using a partial reconstruction of the time series on selected time-scales. We examine daily data from the Nikkei 225 Stock Average and S&P 500 Composite indices from 1/1/69 to 12/5/00, as plotted in Figure 1, as representative examples.

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[1] The support (or support-width) of a function refers to the length of the interval over which it is non-zero.

2 Discrete wavelet analysis

2.1 Multi-resolution decomposition

The discrete wavelet transform is a two-dimensional decomposition of a time series that is specifically designed to detect local characteristics. The two-dimensionality yields time and time-scale information as a result of the compact support of the analyzing basis functions in both time and frequency. The DWT decomposes a general function, $p(t)$, into the form

$$p(t) = \sum_k c_{m0,k}\phi_{m0,k}(t) + \sum_{m\geq m0}^{M} \sum_k d_{m,k}\psi_{m,k}(t), \qquad (1)$$

where the coefficients $\{c_{m0,k}\}$ and $\{d_{m,k}\}$ represent the similarity of the function to their respective basis functions $\phi_{m0,k}$ and $\psi_{m,k}$. The indices m and k indicate the time-scale and time position of the functions respectively. The first sum in the equation represents the trend of the time series and the second, the addition of cycles or detail about the trend at increasingly smaller time-scales.

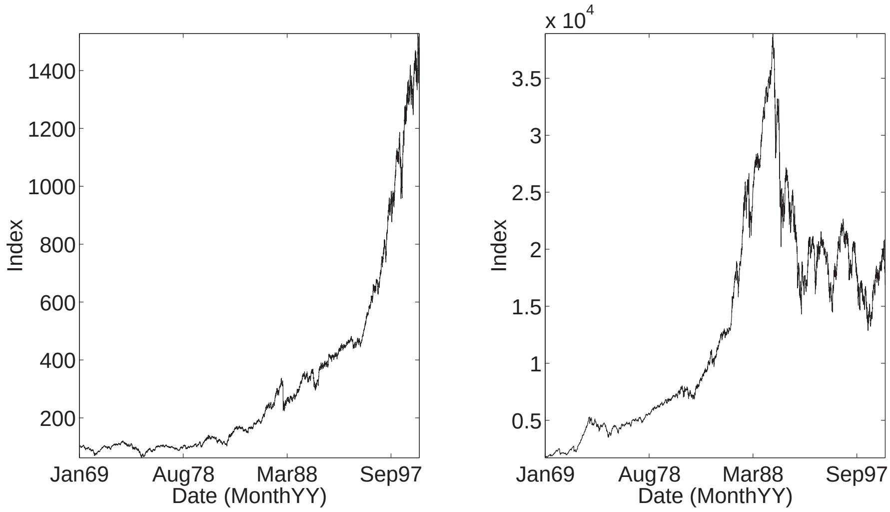

Fig. 1. S&P 500 Composite and Nikkei 225 stock average data from Jan69 to May00.

2.2 Function properties and coefficients

The DWT of a signal is generally expressed by the equations

$$c_{m0,k} = \int p(t)\phi_{m0,k}(t)\mathrm{d}t, \tag{2}$$

$$d_{m,k} = \int p(t)\psi_{m,k}(t)\mathrm{d}t, \tag{3}$$

and has the special property that the functions $\psi_{m,k}$ are all dilations and translations of a single function, $\psi(t)$, referred to as the mother wavelet, specifically,

$$\psi_{m,k} = 2^{m/2}\psi(2^m t - k). \tag{4}$$

Similarly, for the so-called scaling function, $\phi(t)$, we write $\phi_{m,k} = 2^{m/2}\phi(2^m t - k)$. The functions satisfy the following conditions, $\int \phi(t)\mathrm{d}t = 1$ and $\int \psi(t)\mathrm{d}t = 0$, the latter of which ensures the oscillatory nature of the mother wavelet, although in general we also restrict its compact support. Additionally, we impose our wavelet function to have N vanishing moments. A wavelet is said to have N vanishing moments if

$$\int_{-\infty}^{+\infty} t^n \psi(t)\mathrm{d}t = 0,\ n = 0, 1, \ldots, N-1. \tag{5}$$

This allows the removal of polynomial trends of degree $N-1$ from a given data set.

2.3 Filter bank structure

An efficient implementation of the DWT, referred to as *Mallat's Pyramid Algorithm* allows the discrete wavelet transform to be executed in a recursive filter bank structure with $\mathcal{O}(N)$ operations. Writing equation (1) in the form

$$p(t) = A_{m0}(t) + \sum_{m \geq m0}^{M} D_m(t), \tag{6}$$

we obtain an expression for the original time series as a linear superposition of time series which represent outputs from a set of orthogonal band-pass filters.

M corresponds to the highest resolution and thus the smallest time-scales of the analysis, and $D_M(t)$ represents the output of the corresponding band-pass filter. From the construction of equation (4) we see the inherent dyadic (*i.e. power of two*) structure. Consequently, the output of the filter banks contain periods (*i.e.* time-scales) of $2-4$ time units, $4-8$, $8-16$, $\ldots$ and so forth, the time unit being days here.

3 Self-affinity and wavelet variance

A random process $p(t)$ is said to be self-affine with parameter H if for any $a > 0$ it obeys the scaling relation

$$p(t) \triangleq a^{-H} p(at), \tag{7}$$

where $\triangleq$ denotes equality in a statistical sense and H is commonly called the Hurst exponent of the series. Processes with $0 < H < 1$ are described as fractional Brownian motions and for $-1 < H < 0$ we discuss fractional Gaussian noises. $H = \frac{1}{2}$ is the special case of classical Brownian motion and $H = -\frac{1}{2}$ that of Gaussian white noise. This self-affinity is manifest in the power spectrum, $S(f)$, of the process in the form of a power law relationship which yields the general descriptive term $1/f$ process. More precisely,

$$S(f) \propto \frac{1}{|f|^{\gamma}}, \tag{8}$$

and with $\gamma = 2H + 1$, we verify that for white noise $\gamma = 2(-\frac{1}{2}) + 1 = 0$, imposing a flat spectrum.

Consider a function $p(t)$ as a *1/f* process with spectral component γ and let $\{d_{m,k}\}$ be the set of coefficients given by the wavelet transform of $p(t)$ using a wavelet, $\psi(t)$, with

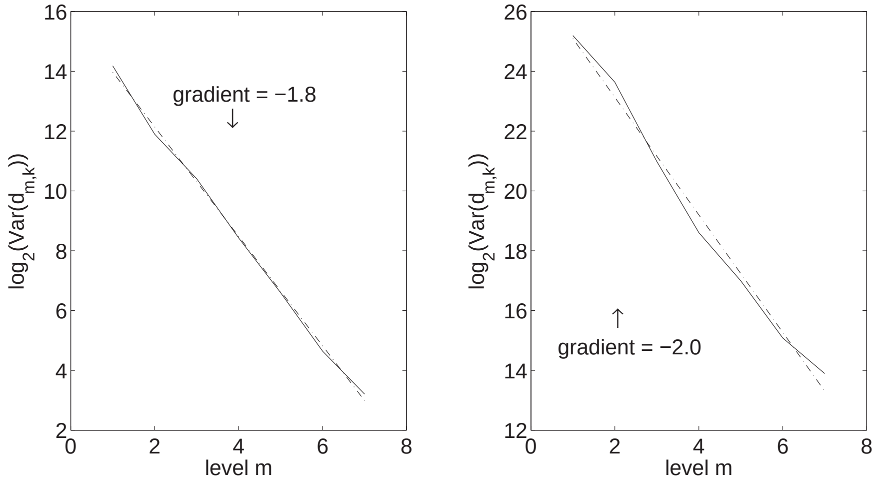

Fig. 2. Log-variance plots for S&P 500 and Nikkei 225 data.

N vanishing moments. Provided $0 < \gamma < 2N$, we find

$$Var(d_{m,k}) \propto 2^{-m\gamma}, \tag{9}$$

where $Var(\cdot)$ denotes the variance [3]. Hence, plotting $log_2(Var(d_{m,k}))$ *vs.* m yields a straight line of gradient $-\gamma$ for the given time series. Classical Brownian motion has spectral parameter $\gamma = 2$ so we obtain a gradient of -2.0. We adopt a symmlet wavelet with 3 vanishing moments for our analysis [4]. Deviations in these log-variance plots have been shown to highlight structure embedded in noise processes, such as periodicities and solitons [2]. More specifically, if energy-concentrating features, *i.e.* coherent structures, are present at a given scale, that scale will often correspond to an increase in the variance characteristics.

4 Results

Figure 2 shows the log-variance graphs of the data used in Figure 1 on time-scales of up to 1 year[2]. We find in both cases that the data scales in a similar fashion to data whose spectra follow a power law, though with fluctuations about the best fit straight line. For the S&P data, a γ value of 1.8 corresponds to a Hurst exponent of 0.4 indicating a degree of anti-persistence on these time-scales.

The Nikkei data exhibits larger variations about the best fit straight line with $\gamma = 2.0$, or $H = 0.5$, corresponding to Brownian motion. We have positive deviations in variance for $m = 1, 2$ and 7. $m = 7$ contains the highest frequency movements, including noise, and the reconstruction on that time-scale includes the largest gains and losses over 2-4 days. $m = 1$ and 2 correspond to time-scales of 128-256 days and 64-128 days *i.e.* approximately 6 months to 1 year and 3–6 months respectively. In Figure 3 we reconstruct the time series on these time-scales, using equation (6), and look at the largest amplitude fluctuations which are responsible for the increase in variance. The dominant recurrent (therefore coherent) structures in the lead up to the major 1990 turning point are the distinct one year cycles indicated by the first pair of parallel lines in Figure 3c. In 3b, as we move through 1990, a repeated six month cycle is present from the *double fall*, the second of which is also evident in 3c at the start of a decreasingly volatile period. These large amplitude cycles result in deviations from a power

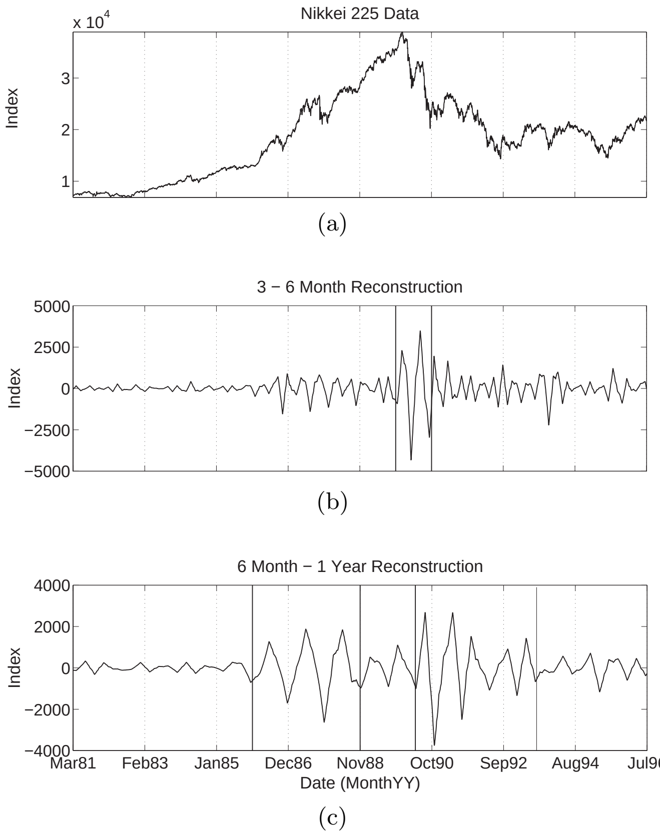

Fig. 3. (a) Nikkei data, (b) and (c) partial reconstructions.

[2] We are restricted to this upper time-scale limit due to the robustness of the wavelet coefficient variance estimator. We are currently looking into better estimation techniques.

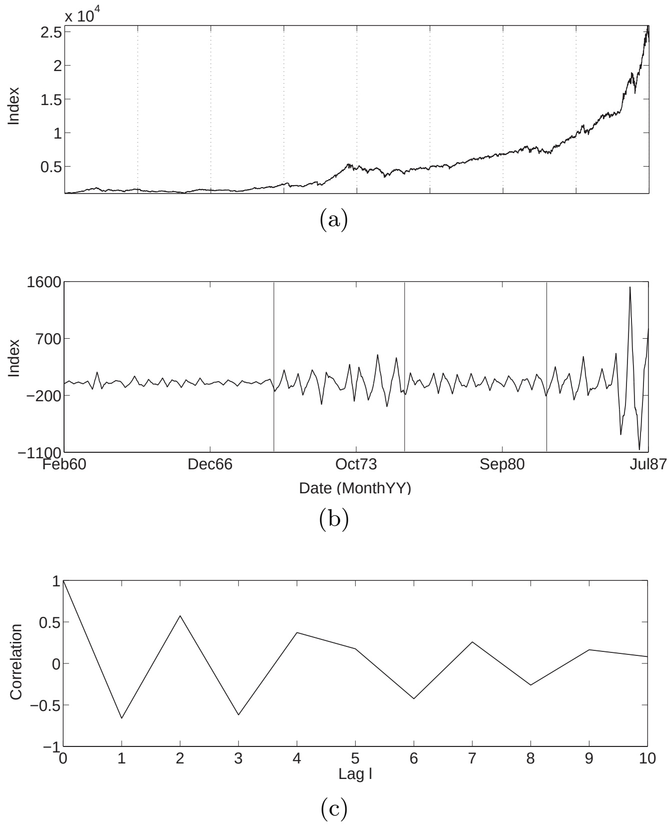

Fig. 4. (a) Nikkei data, (b) reconstruction and (c) autocorrelation.

law spectra model delineated by a linear relationship in the log-variance plot. A similar scaling structure is found for the data shown in Figure 4a, preceding the October 1987 crash, with a positive deviation in the log-variance plot on the 6 month to 1 year time-scale ($m = 1$). A Hurst exponent of 0.6 reflects persistence in the data. 4b shows the time series reconstruction on these time-scales and 4c, the autocorrelation of the corresponding wavelet coefficients. The autocorrelation function does not exhibit the typical fast decay that we expect for *1/f* processes, indicating some coherence between structures. From Figure 4b, we observe that the largest fluctuations on these time-scales occur, firstly, in the years preceding and through 1973, leading into a recessionary period and, secondly, in an increasingly volatile number of years preceding the October 1987 crash including the first of the three clear aforementioned one year cycles.

5 Conclusion

Discrete wavelet analysis was used to investigate self-affinity and the possible presence of coherent structures on time-scales of up to 1 year in samples of Nikkei 225 and S&P 500 data. The data was found to scale in a similar fashion to a *1/f* process though with deviations about the best fit straight line in the log-variance plot. An increase in the variance characteristics and correlation between wavelet coefficients were found to point to large fluctuations, including some cyclic behavior, on time-scales of between 3 months and 1 year. The large amplitude variations correspond to volatile periods through 1973, 1987 and post 1990 and account for the deviations from a $1/f$ type variance structure.

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